A Comparison of Three Versions of Ranked Choice Voting using Computer Simulations*

by Spencer Katzman

ABSTRACT. This paper compares three versions of Ranked Choice Voting (RCV) that differ by their elimination methods: Least First-Place votes, Most Last-Place votes, and Lowest Borda Count. Results are compared to two established measures of majority support—the Condorcet winner and Condorcet loser criteria—and two other criteria that are important to social choice scholars—Independence of Eliminated Alternatives and Reversal Symmetry. R simulations under an Impartial Culture assumption are run for various combinations of candidates and voters. Results show that the Lowest Borda Count outperforms the other two elimination procedures on the Condorcet winner, and Independence of Eliminated Alternatives criteria, while the Most Last-Place method tends to perform at least as well as Borda elimination on reversal symmetry. All three methods perform relatively equally on the Condorcet loser criterion. These findings underscore the need for policymakers and electoral designers to consider the nuances of RCV before adopting the system.

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Introduction

Ranked Choice Voting (RCV)[1] has gained significant momentum in recent years, both in practice and advocacy, emerging as a popular alternative to traditional voting systems. According to the Ranked Choice Voting Resource Center, RCV is now used in major U.S. cities such as San Francisco and has even been adopted statewide in Alaska for electing Senators and allocating Presidential electoral votes. Dougherty and Edward (2001) note that internationally, RCV is used to elect members of the Australian House of Representatives, the national parliament of Papua New Guinea, some mayors in New Zealand, and the President of Ireland. As RCV spreads, interest in its mechanics and fairness has also grown.

In RCV elections, voters rank candidates, and first place votes are tallied based on these rankings. If no candidate earns a majority of first-place votes, the "least desirable" candidate(s) is eliminated using an elimination procedure. Once a candidate(s) is eliminated, a new search for a majority winner commences among the subset of remaining candidates. The process continues until a majority winner is identified with the candidate with a majority of first place rankings winning the election.

This paper compares three different ways of identifying the "least preferred" candidate(s). These elimination rules are Least First Place (LFP), Most Last Place (MLP), and Lowest Borda Count (LBC). The most wildly used method is LFP. It is used in Alaska and San Francisco and it is supported by rank choice advocacy groups like FairVote.org. Proposed in the 19th century, MLP, also known as Coombs method. has been used to select government officials (Grofman et al. 2004) and interestingly, is like the method for eliminating candidates on the

[1] RCV is often used interchangeably with the term Instant Runoff Voting (IRV). Although there are differences that emerge when multiple candidates are being selected, RCV and IRV are identical when they are designed to pick a single winner.

reality television show *Survivor*. Like MLP, LBC has been used sparingly. Nanson (1882) mentions its historical use by the Trinity College Dialectic Society and a variant was also used in Marquette, WI in the 1920s (Gruber). Despite its limited usage, LBC has become an important focal point in United States Presidential elections due to the recent work of eminent economist Amartya Sen and Nobel laureate Eric Maskin (2023) who have argued that it should replace plurality rule and the Electoral College.

Interestingly, Maskin and Sen (2017b) initially advocated for LFP only to back LBC later (2017a). It is that change in their stance that motivated my research as Maskin's rationale was limited to intuitive arguments and a deeper search into the differences in elimination rules seemed appropriate. That deeper search includes comparing the three different elimination procedures in terms of four traditional voting criteria - Condorcet Winner, Condorcet Loser, Independence of Eliminated Alternatives (IEA), and Reversal Symmetry.

Maskin argues that RCV is a viable alternative because it focuses on finding a majority winner, a desirable property in a democracy. For this reason, I evaluate it using the Condorcet winner and loser criteria because, as Nurmi (1987) argues, these criteria "align closely with democratic principles and serve as benchmarks for evaluating the rationality and fairness of voting methods." I include Reversal Symmetry and IEA because they are common measures of democratic fairness advocated by social choice scholars. Specifically, Saari (1995) argues for the importance of Reversal Symmetry because an election that would elect the same candidate if preferences were completely inverted violates basic common sense. He argues it also suggests the voting method is, "sensitive to the structure of rankings in a distorted way, not truly reflecting voter intent; a sign that the method is fundamentally flawed or manipulative." IEA violations can be important indication of the spoiler effect which advocates often claim RCV is designed to avoid (McCune and Wilson (2023). An important example of an election “spoiler” is provided by Jenkins and Morris (2006) who argue that Breckenridge’s late entry into the Presidential election of 1860 led to Abraham Lincoln defeating Stephen A. Douglas, forever changing American history.

The treatment of ties in RCV elections is critical. Contrary to other work that generally assumes elimination ties are broken at random. I do not break ties. Instead, I remove all candidates that tie from the RCV rankings. At times, this can lead to the removal of all remaining candidates which means the election does not produce a winner. Interestingly, under this “non-tie breaking rule”, I show that it is possible that even if there is a winner, it may be a Condorcet loser with LFP and MLP elimination, something not possible when ties are randomly broken. But I prove that a Condorcet loser can never win with LBC elimination even with unbroken ties, a finding that supports LBC.

The paper is organized as follows. First, I review the literature, pointing out how my research and results fit with work that spans more than a century. Next, I review the assumptions made in the paper, the RCV voting rules considered, and the criteria by which results are judged. After providing details on the simulation process, I present my results, including a discussion of the propensities of each elimination rule to produce no winner results, as well as the frequency with which each rule violates the four criteria. Conclusions are then offered as are possibilities for future research. The paper concludes with an appendix that provides the probability of violating the criteria when the no winner results are removed.

Literature Review

Ranked Choice Voting (RCV), also known as Instant Runoff Voting (IRV), has been a central topic in social choice theory and democratic stability. This review synthesizes key academic contributions to the analysis of ranked voting systems, beginning with classical foundations and proceeding to contemporary computational methods. It situates my own research within this literature, contributing new insights into ranked voting systems, particularly regarding elimination and tie breaking rules.

The foundation of ranked voting theory can be traced to Condorcet (1785), who introduced the concept of a Condorcet winner—a candidate who would defeat every other candidate in head-to-head matchups. Condorcet criticized voting mechanisms, including ranked systems, for failing to consistently select this candidate. He also introduced the related concept of the Condorcet loser—the candidate who would lose to every other in pairwise comparisons.

A precursor to using a Borda count elimination rule was first proposed by Nanson (1907). Known as Nanson's rule, all candidates with not more than the average number of Borda votes are eliminated, and the process continues until only one candidate remains. An important caveat is that if all the uneliminated candidates have the same total count then one of them is elected according to a pre-determined method for breaking ties and is declared the winner rather than eliminating all at once and having no winner. Also note that this method differs from typical RCV elections, which eliminate the candidate with the fewest first-place votes at each stage. Fishburn (1977) considered a variant of Nanson's method that is more directly aligned with the rule examined in this paper where just the candidates with the lowest Borda count are eliminated, unless all are tied in which case a winner is randomly selected as in Nanson's rule.

The idea of a tiebreaker being needed during the elimination process is central to RCV outcomes but has been an afterthought in the analysis. Felsenthal and Nurmi (2019) admit existing studies do not include any routine to break ties but mention that in practice ties can be broken in several ways—randomly, based on lexicographic rules using last names, non-anonymously (e.g., the chair breaks ties on a committee), or non-neutrally (e.g. the incumbent candidate is given the advantage).

The limited analysis leads me to investigate the role of tiebreaking rules. I consider an alternative way of handling ties where all tying candidates are eliminated, even if doing so results in no one winning the election. I show that in such a case, a Condorcet loser can actually win an RCV election with LFP or MLP elimination rules. This possibility arises because all other candidates might be eliminated in the penultimate elimination round, leaving the Condorcet loser to win unopposed. I prove that this is not true with LBC because a Condorcet loser can never have a higher Borda count than that of all the other candidates if those candidates are tied.

Maskin and Foley (2002) further explore the implementability of ranked voting systems, showing that RCV, when using the least first-place rule, can fail to be Pareto efficient. Later, Maskin and Sen (2014) expanded Condorcet's critiques by arguing that RCV violates the Independence of Eliminated Alternatives (IEA) criterion. Although their argument is informal, my simulations confirm their intuition: all three elimination rules in RCV violate IEA, and I estimate a non-trivial likelihood of such violations under each.

Tideman (2006) and Benoit (2007) explore RCV's tendency to overlook pairwise victories, raising concerns about non-monotonicity—where a candidate may be harmed by receiving additional support. Foley and Maskin (2022) revisited these ideas in light of Alaska's 2022 RCV election, suggesting that changing the elimination rule from Least First Place to

Lowest Borda Count could help ensure a Condorcet winner is not eliminated. My simulations support their claim.

Examination of criteria violation has been performed theoretically by Lepelley (1993) and Gehrlein (2002) for example and computationally using Monte Carlo simulations by Lepelley et al. (2000) and Merrill (1984). More recent computational analyses, such as Dougherty and Edward (2012) examine the ability of unanimity and majority rule to produce Pareto superior and Pareto optimal alternatives in a two-dimensional spatial voting model. Alternatively, Dougherty and Heckelman (2020) use simulations to calculate the likelihood that various preference aggregation rules violate Arrow's (1950) conditions. Most closely related to my simulation is Dougherty and Edward (2001) who examine several voting procedures including IRV on a single dimension. Their version of IRV uses the common LFP elimination method as a base of comparison to non-RCV type formats. My simulations extend this work to an unrestricted domain and different RCV elimination procedures but do not consider other methods examined by Dougherty and Edward such as plurality, majority rule with a runoff, and Borda Count.

In summary, the literature underscores both the strengths and weaknesses of RCV. RCV can mitigate the spoiler effect and promote majority-backed winners, but it also struggles with criteria like Condorcet consistency, monotonicity, and IEA. My research adds to this body of work by using computational methods to evaluate three RCV elimination rules—LFP, MLP, and LBC—against the Condorcet winner and loser, IEA, and reversal symmetry criteria. The results offer new insights into how elimination rules affect the likelihood of violating core principles in settings ranging from small committees to national elections.

Assumptions

Throughout this paper the term voter is used specifically to mean a citizen who is eligible to vote and who turns out to vote. The term candidate means an individual or option that is on an election ballot as a choice for a voter. In my simulations, there are A $(= 3, 5, \text{or } 9)$ candidates and N $(= 3, 99, 9999)$ voters in an election. By examining a broad mix of candidates and voters, I can provide insight into RCV's performance in small committee settings all the way to presidential elections that are the main concern of Maskin and Sen (2017a, 2017b). Note that the number of candidates and voters is odd, this lessens the possibility of a tie when looking for a majority winner without loss of generality. The number of candidates is limited to nine in that single winner elections rarely have more candidates than that. The number of voters is limited to 9,999 because any increases in N lengthens run time of the simulations without changing the results.

Each candidate draws their preference for candidates under the Impartial Culture (IC) condition, a concept used in social choice theory and voting behavior analysis to model different assumptions about how voter preferences are distributed. Under IC, every possible strict ranking of the candidates is *equally likely*, voter preferences are *independent* of each other, and the population of voter preferences is *uniformly distributed* over the space of all possible rankings. The full profile of drawn voter preferences is referred to as the *preference profile* in an election. The analysis assumes that all voters rank all candidates sincerely and that there is no strategic voting which implies that a profile is an accurate representation of the votes that are cast in an election.

Voting Rules

The RCV process described in the introduction hinges on the elimination procedure used when there is not an immediate majority winner. The following three elimination procedures are considered.

Elimination Rule 1. *Least First Place (LFP)*: The candidate with the least number of first place votes is eliminated. LFP is the standard elimination rule used in practice, including Alaskan Senate and Presidential races.

Elimination Rule 2. *Most Last Place (MLP)*:The candidate with the most last place votes is eliminated. MLP has been used sparingly in the real-world and has usually been confined to relatively small settings such as committee decision making votes (Grofman et al. 2004).

Elimination Rule 3. *Lowest Borda Count (LBC)*: The candidate with the lowest Borda count is eliminated. To calculate a Borda Count, each ballot with A candidates is examined and the candidate with the highest ranking on a ballot receives $A - 1$ points which reflects that they are preferred to that number of other candidates by that voter. Similarly, the second ranked candidate receives $A - 2$ points, and so on until the lowest ranked candidate receives zero points because they are not ranked ahead of any other candidates. The points earned by a candidate are then summed across all ballots. Whichever candidate(s) has the lowest total sum is eliminated in that round.

Voting Criteria

As stated above, four criteria—Condorcet Winner, Condorcet Loser, Independence of Eliminated Alternatives, and Reversal Symmetry—will be used to compare the three RCV elimination rules. The four criteria are:

Criterion 1. *Condorcet Winner criterion*. If a Condorcet Winner exists, it is the candidate who would beat every other candidate in head-to-head matchups using simple majority rule. A voting rule violates the Condorcet Winner criterion when it fails to elect the Condorcet winner, if there is one. No judgement is made regarding this criterion if there is not a Condorcet Winner.

Criterion 2. *Condorcet Loser criterion*. If a Condorcet Loser exists, it is the candidate who would lose to every other candidate in head-to-head matchups using simple majority rule. A voting rule violates the Condorcet Loser criterion when it elects the Condorcet Loser, if there is one. No judgement is made regarding this criterion if there is not a Condorcet Loser.

Criterion 3. *Independence of Eliminated Alternatives (IEA)*. The IEA criterion is examined by checking whether the election result is changed by removing a non-winning candidate from the ballot.[2] To test this, a winner is identified based on the initial preference profile and then non-winning candidates are removed from the preference profile one at a time and winners are identified in each case. If the winner changes from the original result in any of those cases, IEA is violated. If there is no initial winner, then IEA is violated if removing a candidate produces a winner. It should be noted that this is a restricted form of IEA where comparisons are made by removing a single losing candidate and not all subsets of losing candidates.

Criterion 4. *Reversal Symmetry*. Reversal Symmetry takes the original preference profile and inverts it so that every voter's ranking is reversed. If the winner under the original profile remains the winner after preference inversion, then Reversal Symmetry is violated.

[2] It is important to note that IEA is not about the elimination of candidates in the elimination step of RCV.

Simulations

Simulations were run to compare the three elimination rules. Each trial is run under the *Impartial Culture* (IC) which means that any individual voter's preference is generated at random from the set of all possible preference orders and is independent of all other voters' preferences. To get a feel for the complexity of the analysis note that an election with $A = 3$ candidates and $N = 3$ voters has $A! = 6$ possible preference orders and $6^3 = 216$ equally likely preference profiles. This complexity necessitates simulations because of the mathematical difficulty in theoretically calculating the probability of violating criteria in all but the simplest environments. For example, the violation probabilities for the three candidate, three voter case are derived below theoretically but highlight the mathematical impossibility of deriving similar results for combinations of more candidate and/or more voters. Still, the Law of Large Numbers suggests that simulations with adequately many trials will provide good approximations of these unobtainable theoretical probabilities. To that end, 10,000 trials are run using R for the nine different combinations of $A = 3, 5, 9$ candidates and $N = 3, 99, 9999$ voters.

R code is written to do the following for each of the $T = 10{,}000$ trials:

1) Randomly draws a preference profile over A candidates for each of N voters.

2) Searches for a Condorcet Winner and Loser in that preference profile. Record the finding including winner/loser identity.

3) Runs the RCV procedure on the preference profile for the three elimination rules separately and records the winner's identity or denotes no winner in such a case.

4) Removes each losing candidate one at a time. For each elimination, the ranked-choice voting procedures are repeated on the modified set of candidates. The script then examines if the

winner matches the original winner and notes that it is not a violation. If not, it is recorded as a violation of IEA.

5) Re-runs the RCV procedure with preferences reversed and find/record the winner/no winner for the three elimination rules. If the winner is the same, the procedure is counted as violating the reversal symmetry criterion.

6) Use the recorded results to check for violations of each criterion and keep a count thereof.

7) Removes all no winner cases and check the filtered results for violations of the four criteria and keep a count thereof.

Results

This section presents and discusses the simulation results. I begin by examining each elimination method's propensity to generate a no winner outcome.

Table 1: Instances of No Winner Due to Tie

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	2,229	4,770	6,951
Most Last Place	533	176	302
Lowest Borda Count	533	790	959
$N = 99$			
Least First Place	93	126	142
Most Last Place	94	131	128
Lowest Borda Count	33	51	95
$N = 9{,}999$			
Least First Place	1	1	3
Most Last Place	0	1	1
Lowest Borda Count	1	0	1

Notes: Figures represent the number of trials (out of 10,000) where the elimination rule did not result in a winner. This happens in LFP when there is no majority winner in an RCV round and all remaining candidates receive an equal number of first place votes. It happens in MLP when there is no majority winner in an RCV round and all remaining candidates receive the same number of last place votes. It happens in LBC when there is no majority winner in an RCV round and all remaining candidates have the same Borda counts. It is important to note that the mere presence of a tie does not result in a no winner situation. It only does so if ALL remaining candidates are tied.

With only three voters, MLP weakly produces the least no winner results. The intermediate case of 99 voters shows that no winner results become increasing likely as the number of candidates increases although all rules produce a winner in at least 98.5% with the LBC rule producing the least number of no winner outcomes. When there are 9,999 voters the elimination methods become virtually identical with all rules generating no winner outcomes, but all provide near zero chance of producing a no winner result. Fixing N at 3 or 99, as candidates are added, both LFP and LBC experience increases in no winner results. MLP provides the most interesting result as it displays a non-monotonicity in violations for MLP when $N = 3$.

Perhaps the most interesting result in Table 1 is that the Least First Place elimination rule produces far more no winner results compared to the other elimination rules when the number of voters is small. This is related to ties in eliminations. Although all elimination rules result in ties during the process, it is only when a tie occurs between all remaining candidates during elimination that a no winner result occurs. Under IC, it seems that a tie for first place should be as likely as a tie for last place – and it is. However, there is still a difference in no winner outcomes because all elimination rules care about ties at the top because of the majority requirement for a winner. However, MLP and LBC care about other rankings for elimination while LFP only cares about ties at the top. The following examples expand on this idea for $A = N = 3$.

In this case, MLP and LBC only result in no winner when each candidate has one first, one second, and one third place ranking like in the following preference profile.

Preference Profile Example 1 ($A = 3, N = 3$)

Voter 1	Voter 2	Voter 3
A	B	C
B	C	A
C	A	B

Here, there is no majority winner, and all three rules immediately eliminate all three candidates. This is the only way that MLP and LBC can result in such a tie when $A = N = 3$. But LFP can result in a tie in more situations when MLP and LBC do not as can be seen in the following example.

Preference Profile Example 2 ($A = 3, N = 3$)

Voter 1	Voter 2	Voter 3
A	B	C
B	A	A
C	C	B

Here, there is still no majority winner and LFP still eliminates all three candidates immediately. However, MLP and LBC only eliminate candidate C, leaving candidates A and B in a second round where candidate A obtains a majority and wins. This type of case is what adds to the no winner total of LFP (2,229 of the 10,000) whereas the MLP and LBC no winner totals are much lower (533).

From the Impartial Culture (IC) perspective, the preference profile in Example 2 is just as likely as the preference profile in Example 2R, where the preferences are exactly reversed. Since Example 2 has a three-way tie at the top and Example 2R has a three-way tie at the bottom, one might expect both to have no winner results. But this is not the case, and it is at the heart of why the LFP no winner total (2,229) is so much higher than the others (533).

Preference Profile Example 2R ($A = 3, N = 3$)

Voter 1	Voter 2	Voter 3
C	C	B
B	A	A
A	B	C

First, if Example 1 is reversed, none of the three elimination procedures produce a winner because there is not a majority winner and because of the three-way tie for first and for last, every candidate is eliminated by each procedure. The election ends with no winner. However, the same is not true if Example 2 is reversed as in Example 2R.

Here there is a three-way tie for last place, but it does not affect the outcome since there is an immediate majority winner, and the elimination stage is never reached. Therefore, despite the equal likelihood of Examples 2 and 2R under IC, the no winner result is not equally likely because MLP needs both a tie among all three candidates at the bottom and no clear winner at the top. If at least two voters support the same candidate, MLP picks a winner even if there is a tie at the bottom. Hence, the only case in which MLP produces a tie for $N = 3$, $A = 3$ without a winner is the case in which there is a three-way tie at the top and a three-way tie at the bottom whereas LFP only requires a three-way tie at the top for a no winner result.

In fact, in the case of $A = N = 3$, the theoretical probability of a no winner situation happening can be calculated for all three rules using the Fundamental Counting Principle. In all situations, the denominator of the ratio is the total number of possible preference profiles amongst the $N = 3$ voters. Since there are $A = 3$ candidates, there are six (3!) possible orderings from which one is selected for each voter. This means that there are 6^3 possible preference profiles that might be chosen in each iteration of the simulation.

Since ties only occur in MLP and LBC when each candidate has exactly one vote at each ranking, the numerator in the ratio is the same for MLP and LBC. That numerator is $(6)(2)(1) = 12$. This is because once one of the six orderings is chosen for the first voter, there are only two orderings that can be chosen for the second voter that do not give any candidates multiple rankings at a given position. Once the ordering is chosen for the second voter, there is only one ordering possible for the third voter that maintains the completely balanced ranking in Example 1.

The numerator for LFP will be larger than the numerator for MLP and LBC because it will contain all the preference profiles in the MLP and LBC numerators plus those as in Example 2. The numerator in LFP is $(6)(4)(2) = 48$, four times more than in MLP and LBC. Thus, the

Fundamental Counting Principle gives the probability of a no winner outcome in MLP and LBC as $\frac{(6)(2)}{6^3} = 0.055$ and $\frac{(6)(4)(2)}{6^3} = 0.222$ in LFP.

As can be seen in Table 1, the trial probabilities match the theoretical probabilities well; $2229 \approx 4(533)$ with the slight difference being attributable to statistical error in the simulations.

Table 2: Violations of the Condorcet Winner Criterion

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.179	0.382	0.564
Most Last Place	0.000	0.000	0.000
Lowest Borda Count	0.000	0.000	0.000
#CW	9,467	8,461	6,998
$N = 99$			
Least First Place	0.051	0.123	0.221
Most Last Place	0.048	0.121	0.203
Lowest Borda Count	0.000	0.000	0.000
#CW	9,107	7,428	5,489
$N = 9{,}999$			
Least First Place	0.035	0.103	0.186
Most Last Place	0.035	0.100	0.199
Lowest Borda Count	0.000	0.000	0.000
#CW	9,163	7,433	5,530

Notes: Figures represent the probability of failing to select a Condorcet Winner given that a Condorcet Winner exists. Cases where no winner is selected by an elimination rule due to a tie are included as possible cases. Cases in which no winner is selected but a Condorcet Winner exists are treated as violations of the Condorcet Winner Criterion. #CW represents the number of trials out of 10,000 in which there was a Condorcet Winner.

Table 2 presents the results relative to the Condorcet Winner criterion. The most substantive result is that the LBC rule never fails the criterion. This is not a surprise, Fishburn (1977) provides a proof and credits Nanson (1907) with first noting that noting that an RCV elimination rule based on the Lowest Borda Count cannot eliminate a Condorcet winner. In fact, this was the motivation for Maskin and Sen (2023) suggesting Presidential election reform based on RCV with LBC.

This may seem surprising given that it is well known that the Borda Count election itself can violate the Condorcet Winner criterion. In that context, it is important to remember that here, the Borda Count is not supposed to choose a top candidate but to eliminate a bottom candidate. Example 3 below highlights this important difference.

Example 3: Borda Count ($A = 3, N = 99$)

Type 1 Voters (55)	Type 2 Voters (44)
A	B
B	C
C	A

In this example, there are 55 Type 1 voters and 44 Type 2 voters, all with rankings of three candidates. Candidate A is the Condorcet Winner beating both B and C by 55 – 44 margins. The Borda Counts for each are 2(55) + 0(44) = 110 for A, 1(55) + 2(44) = 143 for B, and 0(55) + 1(44) = 44 for C. Therefore, B wins the Borda Count election and the Condorcet winner criterion is violated.

Alternatively, in an RCV election with a Borda Count elimination rule, A wins immediately by a 55 – 44 majority and the Condorcet Winner criterion is not violated.
This obviously is just an example and not a proof, but it does show that RCV with a Borda Count elimination rule is different than straight up Borda Count.

The MLP rule also performs well relative to the Condorcet winner criterion when there are only three voters but fails more often when there are more voters. Whereas LFP has a much higher chance of a violation because of the greater chance of no winner being chosen (even when there is a Condorcet winner)as documented in Table 1.

The case where $N = 3$ provides an interesting comparison. MLP and LBC do not result in violations. This may seem strange at first glance since both MLP and LBC had 533 trials without a winner. However, whenever a preference profile results in no winner, it also does not have a Condorcet winner and there is no violation. But this is not true of LFP which results in no winner outcomes even when there is a Condorcet winner.

Finally, for a fixed number of voters, the number of trials with a Condorcet winner is decreasing in the number of candidates with exception of no violations for MLP and LBC when $N = 3$ mentioned above. Similarly, for a fixed number of candidates, the number of Condorcet winners is decreasing in the number of voters.

Table 3: Violations of the Condorcet Loser Criterion

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.000	0.000	0.000
Most Last Place	0.000	0.000	0.000
Lowest Borda Count	0.000	0.000	0.000
#CL	9,467	8,410	6,934
$N = 99$			
Least First Place	0.004	0.001	0.000
Most Last Place	0.003	0.001	0.000
Lowest Borda Count	0.000	0.000	0.000
#CL	9,107	7,436	5,467
$N = 9{,}999$			
Least First Place	0.001	0.000	0.000
Most Last Place	0.001	0.000*	0.000
Lowest Borda Count	0.000	0.000	0.000
#CL	9,163	7,443	5,492

Notes: Figures represent the probability of selecting a Condorcet Loser given that a Condorcet Loser exists. Cases where no winner is selected by an elimination rule due to a tie are included as possible cases. #CL represents the number of trials a Condorcet Loser was chosen in trials.
*It should be noted that although MLP with $N = 3, A = 5$ shows 0.000 chances of a violation, it did experienced violations, and the apparent perfection is due to rounding.

Table 3 shows performance relative to the Condorcet Loser criterion. Here, all three elimination rules perform very well with no violations for all levels of A when there are only three voters and for all levels of N when there are nine candidates. LBC has the least number of violations overall, never resulting in a Condorcet Loser violation for any combination of A and N. Still, LBC's advantage is minimal as LFP and MLP avoid electing the Condorcet loser in more than 99.5% of trials at worst.

Although small, when $N = 99$, LFP and MLP see a slight uptick in violations compared to $N = 3$ when $A = 3$ and $A = 5$. Interestingly, this uptick is reversed as N increases to 9,999 ($A = 3$ and $A = 5$) and there is a very slight non-monotonicity. This may seem surprising given Nurmi (1987) and others' proofs about RCV never electing a Condorcet Loser. However, their proofs are predicated on all ties being randomly broken, resulting in one elimination per round. When only one candidate is eliminated per round, the ultimate election winner must be the candidate who wins the final two-candidate head-to-head matchup. Since a Condorcet loser loses all head-to-head matchups, one can never when under the random tiebreaking rule.

Recall, however, that ties are not broken in my model. Rather, all tying candidates are eliminated simultaneously. This leads to the new possibility that if a Condorcet loser is amongst the final m candidates and their $m - 1$ opponents all tie for elimination, the Condorcet loser will survive and be the sole remaining candidate and thus the election winner, without ever having to complete in a head-to-head matchup. The proof of the theorem below highlights how this can happen under LFP and MLP elimination rules but not under LBC.

Theorem: A Condorcet Loser can win RCV with LFP and MLP elimination but not with LBC elimination when multiple candidates can be eliminated in a single round.

Proof:

LFP can violate Condorcet Loser criterion:

Proof by counterexample:

Type 1 Voters (5 voters)	Type 2 Voters (3 voters)	Type 3 Voters (3 voters)
C	A	B
A	B	A
B	C	C

With this preference profile, candidate C loses head-to-head to A (6 – 5) and to B (6 – 5), making C a Condorcet loser. In the RCV election, there is no majority winner as A gets 3 first place votes, B gets 3 first place votes, and C gets 5 first place votes. In the first elimination stage, A and B tie with the least first place votes (3 each) and are both eliminated, leaving C as the sole remaining candidate who wins despite being a Condorcet loser, never having to beat another candidate in the majority round. It is worth noting that this is not possible when ties are broken because if only one of A or B were eliminated, the other would go on to defeat C in the majority stage of the next RCV round.

MLP can violate Condorcet Loser criterion:

Proof by counterexample:

Voter Type 1 (1 voter)	Voter Type 2 (4 voters)	Voter Type 3 (1 voter)	Voter Type 4 (3 voters)	Voter Type 5 (2 voters)
C	A	A	B	B
B	C	B	C	A
A	B	C	A	C

With the above preference profile, candidate C loses head-to-head to A (7 – 4) and to B (6 – 5), making C a Condorcet loser. In the RCV election, there is no majority winner in stage one as A gets 5 first place votes, B gets 4 first place votes, and C gets 1 first place vote. In the elimination stage, A and B tie with the most last place votes (4 each) and are both eliminated, leaving C (with only 3 last place votes) as the sole remaining candidate who wins despite being a Condorcet loser. Once again, this is not possible when ties are broken because if only one of A or B were eliminated, the other would go on to defeat C in the majority stage of the next RCV round.

LBC cannot violate the Condorcet Loser criterion:

Proof:

If a Condorcet Loser is to win an RCV election, it must be that all other remaining candidates are eliminated at the same time during an elimination round. Call this number of other candidates m, meaning that there are $m + 1$ total candidates (the others plus the Condorcet loser). Note that $m > 1$ because if $m = 1$, the other remaining candidate would have won the majority round against the Condorcet Loser and there would not have been a subsequent elimination round.

For all other m candidates to be eliminated at once, two things must be true. (1) To be tied, all other m candidates must have the same Borda count (call it K). (2) The Condorcet Loser's Borda count must be higher than K. I will prove these two events are disjoint and therefore, a Condorcet loser cannot win under LBC elimination.

Case 1: N odd.

With $m + 1$ candidates and N voters, there are $Nm(m + 1)/2$ total Borda points available. To be a Condorcet loser, a candidate can at most have $mN/2$ Borda points. This happens if they lose every head-to-head matchup by one vote. This means that among the $\frac{m(m+1)N}{2}$ total Borda points available, there must be at least $\frac{m(m+1)N}{2} - \frac{mN}{2}$ points to be allocated amongst the other m candidates. Since the other m candidates must tie, they each must have $K = \frac{\frac{m(m+1)N}{2} - \frac{mN}{n}}{m} = \frac{mN}{2}$ Borda points.

Since $\frac{mN}{2} = \frac{mN}{2}$, even if the Condorcet loser obtains their maximum number of Borda points, that total cannot exceed the average of the remaining Borda points across the other candidates. Therefore, the Condorcet loser can never have a higher Borda count than that of the tied other candidates and the Condorcet loser can never win the LBC version of RCV when there are an odd number of voters.

Case 2: N even.

In the case where N is even, the maximum number of Borda points a Condorcet winner can have is $m\left(\frac{N}{2} - 1\right)$. Once again, this happens when they lose all head-to-head matchups by one vote. This leaves $\frac{m(m+1)N}{2} - m\left(\frac{N}{2} - 1\right)$ Borda points to be allocated to the other m candidates. Since the other m candidates must tie, they each must have $K = \frac{\frac{m(m+1)N}{2} - m\left(\frac{N}{2} - 1\right)}{m} = \frac{mN+2}{2}$ Borda points.

The maximum possible Borda points for a Condorcet loser, $m\left(\frac{N}{2} - 1\right)$ can be written as $m\left(\frac{N-2}{2}\right)$ and since $\frac{mN+2}{2} > m\left(\frac{N-2}{2}\right)$ it follows that even if the Condorcet loser obtains their maximum number of Borda points, that total cannot exceed the average of the remaining Borda points across the other candidates. Therefore, the other m candidates cannot lose to the Condorcet loser when they are all tied. Therefore, a Condorcet loser can never win the LBC version of RCV when there are an even number of voters.

Since a Condorcet loser can never win the LBC election with an even or odd number of voters, it can never win an LBC election. ∴

Table 4: Violations of the IEA Criterion

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.223	0.477	0.695
Most Last Place	0.053	0.152	0.223
Lowest Borda Count	0.053	0.144	0.249
$N = 99$			
Least First Place	0.136	0.332	0.545
Most Last Place	0.133	0.334	0.528
Lowest Borda Count	0.089	0.213	0.327
$N = 9{,}999$			
Least First Place	0.116	0.307	0.511
Most Last Place	0.115	0.310	0.517
Lowest Borda Count	0.084	0.209	0.328

Notes: Figures represent the probability that a different result occurs if a non-winning candidate is removed from the initial preference profile. Cases where no winner is selected by an elimination rule due to a tie are included as possible cases.

Table 4 shows that LBC generates the least number of IEA violations except for the case of $N = 3, A = 9$ where it is only slightly worse than MLP. LFP performs particularly poorly when $A = 3$ because of the excessive number of no winner outcomes but as N increases to 99 and 9,999, LFP and MLP violation probabilities become very similar. As expected, violation probabilities are increasing in both A and N for all rules. Interestingly, although LBC performs best relative to IEA, this is its worst performance of the four criteria.

Table 5: Violations of the Reversal Symmetry Criterion

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.053	0.238	0.485
Most Last Place	0.053	0.003	0.002
Lowest Borda Count	0.053	0.015	0.013
$N = 99$			
Least First Place	0.032	0.021	0.010
Most Last Place	0.029	0.017	0.005
Lowest Borda Count	0.052	0.014	0.002
$N = 9{,}999$			
Least First Place	0.024	0.017	0.006
Most Last Place	0.023	0.015	0.005
Lowest Borda Count	0.053	0.015	0.002

Notes: Figures represent the probability that the same result occurs before and after preference profiles are reversed. Cases where no winner is selected by an elimination rule due to a tie are included as possible cases.

All rules perform relatively well on the reversal symmetry criterion, and while LFP tends to have the most violations, the comparison of MLP and LBC is mixed. MLP is better than LBC when $A = 3$, they are roughly the same when $A = 5$, and LBC is better than MLP when $A = 9$. The case of $N = 3$ is different than under the other three criteria in cases when $N = 3$. There, the LFP violation is increasing in A, the LBC rate is decreasing A, and the MLP rate is non-monotonic.

The $A = 3, N = 3$ case is interesting because the only instance where the three rules violate reversal symmetry is when each candidate has one of each ranking and there is a tie in all three. In that case, reversing the preference order continues to result in no winner, thus causing the violation.

Conclusions

In general, which voting rule is “best” depends upon the properties valued by a community. Lately, many communities have expressed interest in RCV and begun advocating for its adoption. The results established in this paper suggest it is crucial that policymakers and advocates consider specific details of the election process such as the elimination method and tie-breaking rules. For example, a major advocate of RCV is Fairvote.org which promotes RCV with a Least First Place elimination method and their website does not make any mention of how ties are broken. Once again, these details matter and should be part of any proposal or debate regarding RCV.

This paper provides substantive computational evidence that the Lowest Borda Count elimination is generally better than the Least First Place and Most Last Place rules in terms of frequency of violating the Condorcet winner, Condorcet loser, Independence of Eliminated Alternatives, and Reversal Symmetry criteria. Future work might focus on how random elimination of candidates could change these results. IEA results could also be extended to removals of all subsets of candidates rather than the simple single candidate removals examined here. Finally, the different elimination rules could be compared relative to other important criteria such as monotonicity.

In sum, Maskin’s recommendation for using RCV with Lowest Borda Count elimination in United State Presidential elections provided the motivation for this paper. The results in this paper support the suggested use of LBC if RCV is used because LBC is simply better than the status quo LFP in most instances.

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Appendix

This appendix presents violation findings under the condition that all no winner results are removed from the comparison.

Table A1: Violations of the Condorcet Winner Criterion (conditional)

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.000	0.000	0.000
Most Last Place	0.000	0.000	0.000
Lowest Borda Count	0.000	0.000	0.000
#CW: LFP	7,771	5,230	3,049
#CW: MLP	9,467	8,461	6,998
#CW: LBC	9,467	8,461	6,998
$N = 99$			
Least First Place	0.043	0.114	0.212
Most Last Place	0.040	0.111	0.196
Lowest Borda Count	0.000	0.000	0.000
#CW: LFP	9,029	7,355	5,431
#CW: MLP	9,034	7,342	5,440
#CW: LBC	9,107	7,428	5,489
$N = 9{,}999$			
Least First Place	0.035	0.102	0.186
Most Last Place	0.035	0.100	0.199
Lowest Borda Count	0.000	0.000	0.000
#CW: LFP	9,162	7,432	5,530
#CW: MLP	9,163	7,433	5,529
#CW: LBC	9,163	7,433	5,530

Notes: Figures represent the probability of failing to select a Condorcet Winner given that a Condorcet Winner exists. Cases where no winner is selected by an elimination rule due to a tie are not included as possible cases. #CW represents the number of trials where there was a Condorcet winner, and the elimination rule produced a winning candidate.

Table A1 shows that once trials with no winner are filtered out, all rules perform very well relative to the criterion when there are only three voters. This relates to the intuition discussed regarding Table 2 above where the large number of violations under the LFP rule was driven by the large number of ties under that rule. Once eliminated, LFP performs as perfectly as MLP and LBC. Still, this perfection of both the LFP and MLP rules ceases with 99 and 9,999 voters while LBC continues its perfection, always electing the Condorcet winner.

An important characteristic about Table A1 is that since each rule possibly produced a different number of no winner results, filtering out no winner results leads to a different number of Condorcet winners remaining after the filtering. These differences are noted by the addition of rows #CW: LFP, #CW: MLP, and #CW: LBC.

Table A2: Violations of the Condorcet Loser Criterion (conditional)

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.000	0.000	0.000
Most Last Place	0.000	0.000	0.000
Lowest Borda Count	0.000	0.000	0.000
#CL: LFP	7,771	4,663	2,212
#CL: MLP	9,467	8,234	6733
#CL: LBC	9,467	7,794	6282
$N = 99$			
Least First Place	0.004	0.001	0.000
Most Last Place	0.003	0.001	0.000
Lowest Borda Count	0.000	0.000	0.000
#CW: LFP	9.029	7,340	5,383
#CW: MLP	9,034	7,345	5,403
#CW: LBC	9,107	7,403	5,421
$N = 9{,}999$			
Least First Place	0.001	0.000	0.000
Most Last Place	0.001	0.000	0.000
Lowest Borda Count	0.000	0.000	0.000
#CW: LFP	9,162	7,443	5,491
#CW: MLP	9,163	7,442	5,492
#CW: LBC	9,163	7,433	5,492

Notes: Figures represent the probability of selecting a Condorcet Loser given that a Condorcet Loser exists. Cases where no winner is selected by an elimination rule due to a tie are not included as possible cases. Since the different rules have a different number of trials removed for No Winner (see Table 1), each might have a different number of Condorcet losers, leading to the inclusion of the rows #CL: LFP, #CL: MLP, and #CL: LBC #CL

As with the unconditional case, once no winner results are removed, LBC has the least number of violations, never electing a Condorcet Loser (see Theorem above). But this advantage is negligible as in the unconditional case since LFP and MLP elect non-Condorcet losers more than 99.5% of the time and usually more than even that. As with the unconditional case, there is a non-monotonicity in the violation percentage as N is increased for the $A = 3$ and $A = 5$ trials although the changes were very small. Interestingly, with nine candidates, none of the rules caused a violation.

Table A3: Violations of the Independence of Eliminated Alternatives Criterion (conditional)

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.000	0.000	0.000
Most Last Place	0.000	0.137	0.198
Lowest Borda Count	0.000	0.070	0.169
$N = 99$			
Least First Place	0.127	0.323	0.538
Most Last Place	0.125	0.326	0.522
Lowest Borda Count	0.086	0.209	0.320
$N = 9{,}999$			
Least First Place	0.116	0.307	0.511
Most Last Place	0.115	0.310	0.517
Lowest Borda Count	0.084	0.209	0.328

Notes: Figures represent the probability that eliminating a candidate results in a different winner than the winner under the full slate of candidates. Cases where no winner is selected by an elimination rule due to a tie are not included as possible cases.

Once no winner trials are removed, none of the elimination rules have a violation when $A = N = 3$. This is because all instances without a no winner result had a majority winner and with only two candidates left after an elimination, that winner continues to win, leaving no IEA violations. As A increases with $N = 3$, LFP performs the best of the three rules with LBC performing second best, and MLP last. This is because LFP continues to only produce violations when there are no winner results and with those trials removed, LFP remains perfect in the conditional case. Alternatively, MLP and LBC have other violation sources when $N > 3$.

With $N = 99$ and $N = 9{,}999$, LFP's advantage disappears, and its violation probabilities become virtually the same as MLP while LBC has fewer violation probabilities than both. Except for LFP's lack of violations when $N = 3$, violation probabilities tend to be increasing in the number of candidates. When N increases from 3 to 99, all three rules see a significant jump in violation probability which levels off for $N = 99$ and $N = 9{,}999$ with even a few slight decreases with the larger number of voters.

Table A4: Violations of the Reversal Symmetry Criterion (conditional)

Voting rule	$A = 3$	$A = 5$	$A = 9$
$N = 3$			
Least First Place	0.000	0.000	0.000
Most Last Place	0.000	0.003	0.000
Lowest Borda Count	0.000	0.005	0.001
$N = 99$			
Least First Place	0.032	0.021	0.010
Most Last Place	0.029	0.017	0.005
Lowest Borda Count	0.049	0.014	0.002
$N = 9{,}999$			
Least First Place	0.024	0.017	0.006
Most Last Place	0.023	0.015	0.002
Lowest Borda Count	0.053	0.015	0.002

Notes: Figures represent the probability that the same result occurs in a trial and when the preference profiles are reversed. Cases where no winner is selected by an elimination rule due to a tie are not included as possible cases.

All rules perform relatively well regarding reversal symmetry with the highest failure rate being 0.053 by LBC when $A = 3, N = 9{,}999$. LFP performs better than MLP and LBC where $N = 3$ with both perform similarly. Interestingly, LBC performs the worst when $A = 3$ but is weakly the best when $A = 5$ and $A = 9$ (LFP performs worst in those cases).

As in the unconditional case (Table 5), there is a non-monotonicity with MLP and LBC as A increases when $N = 3$. For the cases of $N = 99$ and $N = 9{,}999$, all probabilities are decreasing in A. There is a similar non-monotonicity with LFP and MLP as N increases for all values of A while the probability is increasing with LBC as N increases for all values of A.